Exercise 7

The table shows the position of a motorcyclist after accelerating from rest.

t (seconds)	0	1	2	3	4	5	6
s (feet)	0	4.9	20.6	46.5	79.2	124.8	176.7

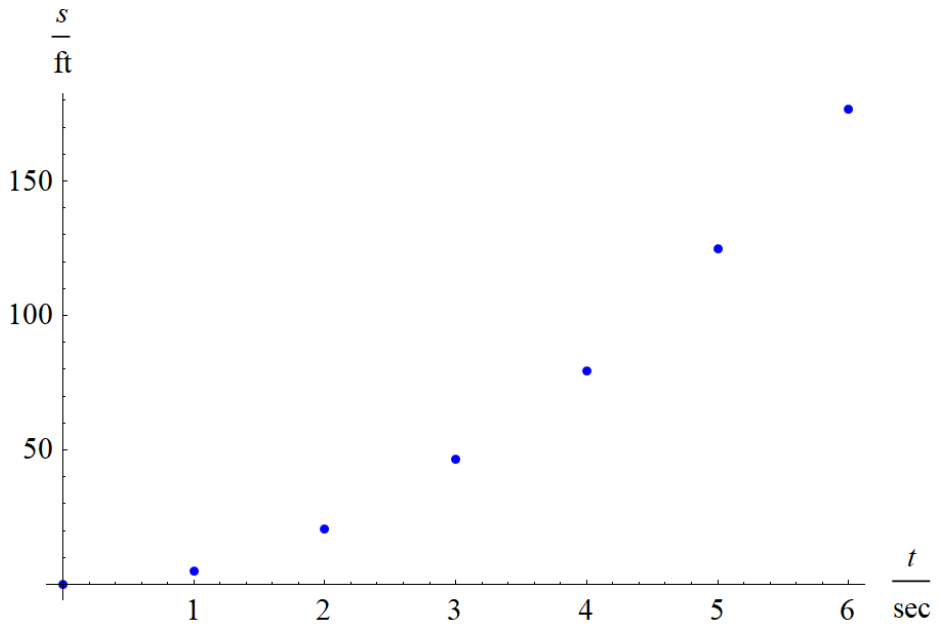
(a) Find the average velocity for each time period:

(i) $[2, 4]$ (ii) $[3, 4]$ (ii) $[4, 5]$ (ii) $[4, 6]$

(b) Use the graph of s as a function of t to estimate the instantaneous velocity when $t = 3$.

Solution

Below is a plot of the motorcyclist's position, using the points in the table: $(0, 0)$, $(1, 4.9)$, $(2, 20.6)$, $(3, 46.5)$, $(4, 79.2)$, $(5, 124.8)$, and $(6, 176.7)$.



The average velocity in each time period is given by the slope of the line going through two points.

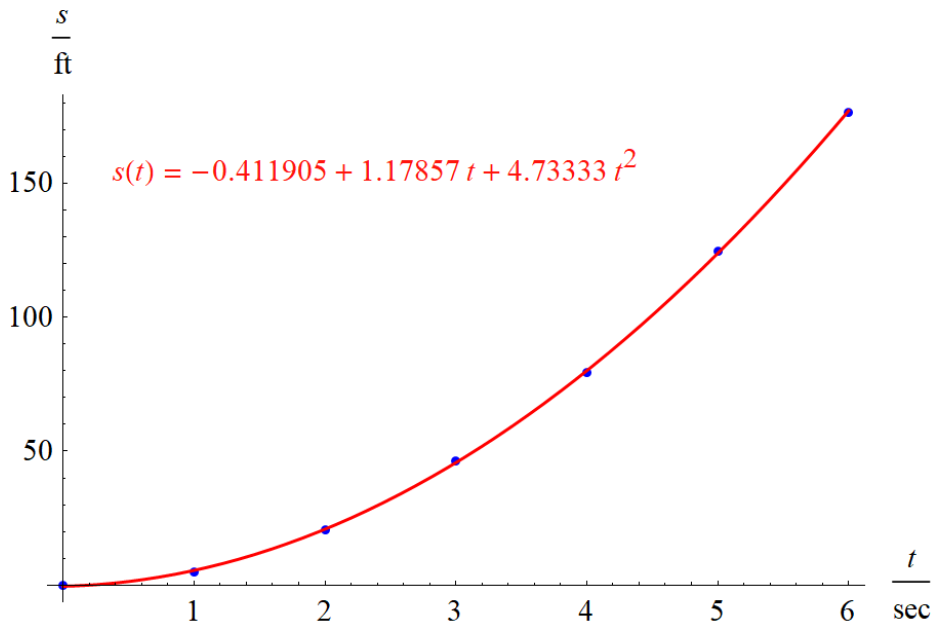
$$[2, 4]: \quad m = \frac{s(4) - s(2)}{4 - 2} = \frac{79.2 - 20.6}{4 - 2} = 29.3 \ \frac{\text{ft}}{\text{s}}$$

$$[3, 4]: \quad m = \frac{s(4) - s(3)}{4 - 3} = \frac{79.2 - 46.5}{4 - 3} = 32.7 \ \frac{\text{ft}}{\text{s}}$$

$$[4, 5]: \quad m = \frac{s(5) - s(4)}{5 - 4} = \frac{124.8 - 79.2}{5 - 4} = 45.6 \ \frac{\text{ft}}{\text{s}}$$

$$[4, 6]: \quad m = \frac{s(6) - s(4)}{6 - 4} = \frac{176.7 - 79.2}{6 - 4} = 48.75 \ \frac{\text{ft}}{\text{s}}$$

Use Mathematica's Fit function to determine the optimal quadratic curve going through the provided data.



The instantaneous velocity at $t = 3$ is

$$\frac{ds}{dt} = 1.17857 + 2(4.73333)t$$

$$\left.\frac{dy}{dt}\right|_{t=3} = 1.17857 + 2(4.73333)(3) = 29.5786 \ \frac{\text{ft}}{\text{s}}.$$